Name:

SM275 · Mathematical Methods for Economics — Fall 2019 · Uhan

Exam 1 – 23 September 2019

Instructions

- You have until the end of the class period to complete this exam.
- You may use your calculator.
- You may consult the SM275 Formula Table given to you with this exam.
- You may not consult any other outside materials (e.g. notes, textbooks, homework, computer).
- **Show all your work.** To receive full credit, your solution must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.

Problem	Weight	Score
1	1	
2	1	
3	2	
4	1	
5	2	
6	2	
7	2	
8	1	
9	2	
10	2	
11	2	
12	2	
13	2	
14	2	
Total		/ 240

For Problems 1-4, consider the following setting.

Suppose we win the lottery. We are given 30 annual payments of $100 each, with the first payment given now. Assume that whenever we get a payment, we put it in a savings account earning interest at an annual rate of 0.02, compounded annually.

Let A_n be the amount in the savings account after n years.

Problem 1. Write the DS for this setting.

Problem 2. Write the IC for this setting.

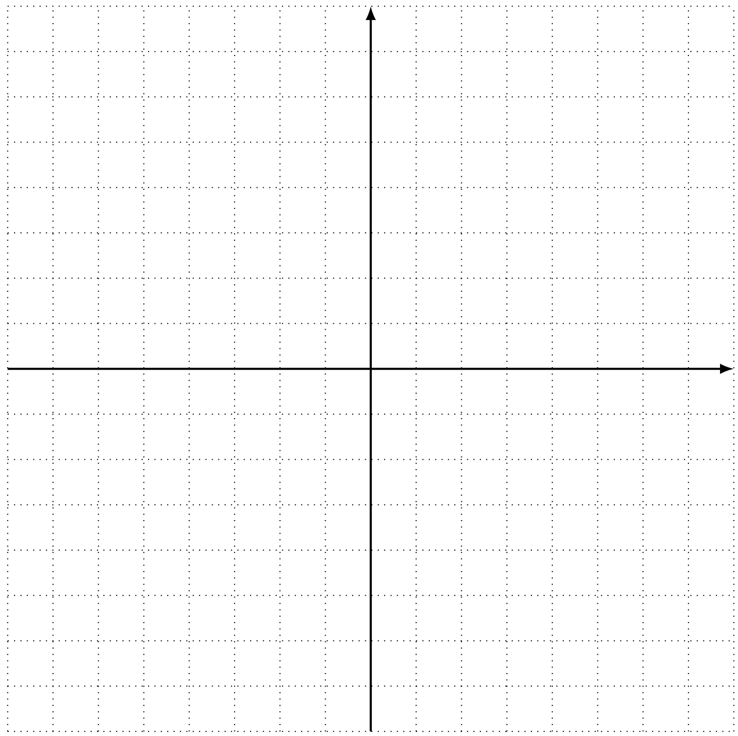
Problem 3. Find the particular solution that satisfies the IC.

Problem 4. Use the particular solution you found in Problem 3 to find the amount in the account after 30 payments.

For Problems 5-7, consider the DS

$$A_{n+1} = 2A_n + 1 \qquad n = 0, 1, 2, \ldots$$

Problem 5. Draw the cobwebs with $A_0 = -3$ and $A_0 = 1$. Don't forget to indicate the direction of the cobwebs.



Problem 6. Explain why the fixed point of the DS is −1.

Problem 7. Is the fixed point −1 attracting, repelling, or neither? Briefly explain.

For Problems 8 and 9, consider the discrete market model

$$D_t = S_t \tag{1}$$
$$D_t = 18 - P_t \tag{2}$$
$$S_t = -2 + 3P_{t-1} \tag{3}$$

where at time t, D_t is the demand, S_t is the supply, and P_t is the price. In addition, suppose $P_0 = 8$.

Problem 8. In words, briefly explain why equation (2) makes sense from an economic perspective.

Problem 9. Using the equations above, find a first order linear DS that describes how the price evolves over time. Your answer should look like: "$P_{t+1} = \ldots$" Do not solve the DS.

For Problems 10 and 11, consider the DS

$$A_{n+2} = A_{n+1} + 2A_n + 6 \qquad n = 0, 1, 2, \ldots$$

Problem 10. Find the general solution.

Problem 11. Find the particular solution satisfying the IC $A_0 = 1$, $A_1 = 2$.

Problem 12. Consider following the national income model, with marginal propensity to consume $m = \frac{3}{4}$ and accelerator $\ell = \frac{1}{3}$:

$$\begin{aligned} T_n &= C_n + I_n + G_n \\ C_{n+1} &= \frac{3}{4} T_n \\ I_{n+1} &= \frac{1}{3}(C_{n+1} - C_n) \\ G_n &= 1 \end{aligned} \qquad n = 0, 1, 2, \ldots$$

where at time n, T_n is the total national income, C_n is the amount of consumer expenditures, I_n is the amount of private investment, and G_n is the amount of government expenditures. We showed that we can rewrite this model as the following DS:

$$T_{n+2} = T_{n+1} - \frac{1}{4} T_n + 1 \qquad n = 0, 1, 2, \ldots$$

Suppose $C_0 = 2$ and $I_0 = 1$. Find the IC for the DS.

For Problems 13 and 14, consider the following DS:

$$A_{n+2} = -\frac{2}{3}A_{n+1} + \frac{1}{3}A_n + 8 \qquad n = 0, 1, 2, \ldots$$

The general solution to this DS is

$$A_n = c_1(-1)^n + c_2\left(\frac{1}{3}\right)^n + 6.$$

Problem 13. Find the fixed point of this DS.

Problem 14. Is the system stable, unstable, or neither? Briefly explain.

Additional page for scratchwork or solutions